PROBABILITY AND STOCHASTIC PROCESS

PAPER- 301

1 mark questions

1. Define a random variable.
2. What is a function of a random variable?
3. What are moments in the context of random variables?
4. Define the moment generating function of a random variable.
5. Can you have a function of several random variables?
6. What does it mean for two random variables to be independent?
7. What is the covariance of two random variables?
8. Define the correlation of two random variables.
9. What is the first moment of a random variable?
10. Define conditional expectation.
11. What is a bivariate normal distribution?
12. What is a multivariate normal distribution?
13. Define the exponential family of distributions.
14. What are some examples of the exponential family of distributions?
15. What is meant by the modes of convergence?
16. State the weak law of large numbers.
17. State the strong law of large numbers.
18. What is a limiting moment generating function?
19. State the Central Limit Theorem.
20. Define random sampling.
21. What are sample characteristics?
22. What is the distribution of sample characteristics?
23. Define the chi-square distribution.
24. Define the T distribution.
25. Define the F distribution.
26. What is an exact sampling distribution?
27. Define a stochastic process.
28. Give an example of a stochastic process.
29. Define a Markov chain.
30. Give an example of a Markov chain.
31. What is the Chapman-Kolmogorov equation?
32. How are states in a Markov chain classified?
33. What are limiting probabilities in a Markov chain?
34. Give an example of an application of a Markov chain.
35. What is the gambler's ruin problem?

36. What is the second moment of a random variable?
37. What does the moment generating function of a random variable tell us?
38. Give an example of an application of the strong law of large numbers.
39. What is the difference between the weak and strong law of large numbers?
40. What does the Central Limit Theorem tell us about the distribution of a large number of independent and identically distributed random variables?
41. What is the difference between the T distribution and the normal distribution?
42. Give an example of a situation where the chi-square distribution is used.
43. What does the F distribution tell us about the variances of two normal distributions?
44. Give an example of a situation where the F distribution is used.
45. Give an example of a function of several random variables.
46. What does the correlation of two random variables tell us about their relationship?
47. What does the covariance of two random variables tell us about their relationship?
48. In what situation would you use an exact sampling distribution?
49. What does the limiting probability of a state in a Markov chain tell us?
50. Give an example of an application of the gambler's ruin problem.
51. A __________ variable is a variable whose possible values are numerical outcomes of a random phenomenon.
52. A function of a random variable is also known as a __________.
53. The mean of a random variable is also known as its first __________.
54. The __________ generating function is a way to characterize the entire probability distribution of a random variable.
55. Two random variables are __________ if the occurrence of one does not affect the occurrence of another.
56. The covariance is a measure of the __________ between two random variables.
57. The correlation is a measure of the __________ and direction of the linear relationship between two random variables.
58. Moments are used to understand the __________ about the mean of a probability distribution.
59. The conditional expectation or conditional mean of an random variable is the expected value of the variable given some __________.
60. A __________ normal distribution is a normal distribution in a two-dimensional random vector.

61. A __________ normal distribution is a generalization of the one-dimensional (univariate) normal distribution to higher dimensions.
62. The exponential family of distributions is a parametric set of probability distributions of a certain __________.
63. In probability theory and statistics, the term __________ of convergence refers to the way in which a sequence of random variables converge to a random variable.
64. The __________ law of large numbers states that the sample average converges in probability towards the expected value.
65. The __________ law of large numbers states that the sample average converges almost surely to the expected value.
66. A __________ moment generating function characterizes the limit of a sequence of moment generating functions.
67. The Central Limit Theorem states that the sum of a large number of independent and identically distributed random variables, each with finite mean and variance, will have approximately a __________ distribution.
68. __________ sampling is a basic sampling technique where we select a group of subjects (a sample) for study from a larger group (a population).
69. __________ characteristics are quantities such as the mean, median, and standard deviation.
70. The __________ distribution is related to the square of a standard normal distribution.
71. The __________ distribution is related to the ratio of the variance of a normal distribution to its degrees of freedom.
72. The __________ distribution is related to the ratio of two chi-square distributions.
73. __________ sampling distributions are the probability distribution of a statistic obtained through a large number of samples drawn from a specific population.
74. A __________ process is a collection of random variables representing the evolution of some system of random values over time.
75. __________ chains are a type of stochastic process that undergo transitions from one state to another on a state space.
76. The __________ equation gives a way of computing the n-step transition probability of a Markov chain.
77. In a Markov chain, states are classified as transient, recurrent, null recurrent, and __________.
78. __________ probabilities in a Markov chain are the probabilities that the chain will be in a particular state after a large number of steps.

79. The gambler's ruin problem is a type of __________ problem involving a gambler with a finite amount of money, who bets iteratively on a fair coin toss.

2 mark questions

1. If X is a random variable, write down the formula for the expected value of X.
2. Suppose Y = aX + b is a function of the random variable X, write the formula for the expected value of Y.
3. Let X be a random variable with E[X] = μ and Var[X] = σ^2. What are E[aX + b] and Var[aX + b] for constants a and b?
4. Write the definition of covariance for two random variables X and Y.
5. If X and Y are independent random variables, what is Cov[X, Y]?
6. Write down the definition of the correlation coefficient for two random variables X and Y.
7. What is the relationship between correlation and covariance?
8. If X and Y are independent random variables, what is their correlation?
9. What is the definition of the conditional expectation E[X | Y = y]?
10. Write the joint probability density function of a bivariate normal distribution.
11. Given that X and Y are normally distributed, how can you tell if they have a multivariate normal distribution?
12. What is the general form of the probability density function of an exponential family distribution?
13. Write down the weak law of large numbers.
14. What is the strong law of large numbers?
15. Write the definition of convergence in probability.
16. What is the limiting moment generating function of a sequence of random variables {X_n}?
17. Write down the Central Limit Theorem.
18. Write the chi-square distribution in terms of Z, a standard normal random variable.
19. Write the T distribution in terms of Z, a standard normal random variable, and V, a chi-square random variable.
20. Write the F distribution in terms of two independent chi-square random variables V1 and V2.
21. Define a stochastic process {X(t), t $\in$ T}.
22. What is the defining property of a Markov chain?
23. Write down the Chapman-Kolmogorov equation for a Markov chain.
24. How are states in a Markov chain classified?

25. Define the limiting probabilities of a Markov chain.

6/7 marks questions

1. Define a random variable and provide examples of situations where random variables can be used.
2. Explain the concept of a function of a random variable and discuss how it differs from an ordinary function.
3. Discuss the concept of moments in random variables. How are moments used in the context of random variables and their distributions?
4. Define and provide examples of the moment generating function of a random variable.
5. Can you have a function of several random variables? If so, provide an example and discuss its applications.
6. Define the concept of independent random variables and provide examples.
7. Discuss the concept of covariance and correlation of random variables. How are they different and how are they used in statistics?
8. Define conditional expectation and provide examples. How is it used in statistics and probability theory?
9. Define the bivariate normal distribution and the multivariate normal distribution. How are they used in statistics and data analysis?
10. Discuss the concept of the exponential family of distributions. Provide examples and discuss its importance in statistics.
11. Discuss the concept of modes of convergence in statistics and probability theory.
12. Explain the weak law of large numbers and the strong law of large numbers. Provide examples and discuss their importance in statistics.
13. Discuss the concept of the limiting moment generating function and its importance in statistics.
14. Explain the Central Limit Theorem and discuss its significance in statistics and probability theory. Provide examples where the Central Limit Theorem is used.
15. Discuss the concept of random sampling and provide examples of situations where random sampling is used.
16. Explain the concept of sample characteristics and their distribution. Provide examples and discuss their importance in data analysis.
17. Define and discuss the chi-square, T, and F distributions. Provide examples and discuss their importance in statistics.
18. Discuss the concept of exact sampling distribution and provide examples. How does it differ from the sampling distribution and where is it used?

19. Define a stochastic process and provide examples of situations where stochastic processes can be used.
20. Discuss the concept of Markov Chains and the Chapman-Kolmogorov equation. Provide examples and discuss their applications in statistics and data analysis.
21. Discuss the concept of classification of states in Markov Chains. Provide examples and discuss their importance in the analysis of Markov Chains.
22. Define and discuss the concept of limiting probabilities in Markov Chains. Provide examples and discuss their importance.
23. Discuss the Gambler's Ruin problem in the context of stochastic processes and Markov Chains. Provide examples and discuss its applications.