DIFFERENTIAL GEOMETRY
PAPER- 302

1 mark questions

1. Define the tangent of a curve in 2D space.
2. What is the contact of two curves?
3. Explain the concept of the osculating plane of a curve.
4. Define the principal normal of a curve.
5. What is the binormal of a curve in 3D space?
6. Define curvature of a curve at a given point.
7. Explain the concept of torsion of a curve.
8. State the Serret-Frenet formula for a curve in 3D space.
9. Define the osculating circle of a curve at a given point.
10. What is the osculating sphere of a curve?
11. Define a ruled surface.
12. What is a developable surface?
13. Explain the concept of a tangent plane to a ruled surface.
14. State the necessary and sufficient condition for a surface f=0 to be developable.
15. Define the metric of a surface.
16. Explain the first fundamental form of a surface.
17. What are the second and third fundamental forms of a surface?
18. Define the Gaussian curvature of a surface.
19. What is an umbilic point on a surface?
20. Define the radius of curvature of a normal section at an umbilic point.
21. Define the normal curvature of a surface at a given point.
22. Explain Meunier's theorem related to normal curvature.
23. Define the lines of curvature on a surface.
24. What are the principal radii of a surface at a given point?
25. State the relation between the fundamental forms of a surface.
26. Define principal directions on a surface.
27. What are the principal curvatures of a surface at a given point?
28. Define the mean curvature of a surface.
29. What is the first curvature of a surface?
30. Explain the concept of Gaussian curvature of a surface.
31. State the relationship between the Gaussian curvature and the principal curvatures of a surface.
32. Define an umbilic point on a surface.
33. What is the radius of curvature of a normal section at an umbilic point Z=F(x, y)?
34. Calculate the radius of curvature at a given section through any point of Z=F(x, y).
35. Define asymptotic lines on a surface.
36. What are the fundamental magnitudes of some important surfaces?
37. Define the concept of orthogonal trajectories on a surface.
38. State the Existence and Uniqueness theorem for curves.
39. Explain the characteristics of a Bertrand curve.
40. Define the involute of a curve.
41. Explain the concept of the evolute of a curve.
42. Define the concept of ruled surfaces.

43. What is a developable surface?
44. Explain the tangent plane condition for a ruled surface.
45. State the necessary and sufficient condition for a surface f=0 to represent a developable surface.
46. Define the metric of a surface.
47. Explain the first, second, and third fundamental forms of a surface.
48. What are the fundamental magnitudes of a surface?
49. Define orthogonal trajectories on a surface.
50. State Meunier's theorem related to the normal curvature of a surface.

2/3 marks questions

1. Define the tangent vector of a curve at a point P in 2D space, denoted as T(P).
2. Given two curves C1 and C2, define the contact of curves C1 and C2 at a common point P, denoted as C1 ∩ C2 at P.
3. Explain the concept of the osculating plane of a curve C at a point P, denoted as $\pi(P)$.
4. Define the principal normal vector of a curve C at a point P, denoted as N(P).
5. What is the binormal vector of a curve C at a point P in 3D space, denoted as B(P)?
6. Define the curvature of a curve C at a point P, denoted as $\kappa(P)$.
7. Explain the concept of torsion of a curve C at a point P, denoted as $\tau(P)$.
8. State the Serret-Frenet formula for a curve C in 3D space.
9. Define the osculating circle of a curve C at a point P, denoted as C(P).
10. What is the osculating sphere of a curve C at a point P, denoted as S(P)?
11. Define a ruled surface S generated by two curves C1 and C2, denoted as S = {P(u, v) = (1-u)C1(v) + uC2(v)\ | u, v \in \mathbb{R}}.
12. What is a developable surface S formed by a family of straight lines, denoted as S = {P(u, v) = C(u) + vN(u)\ | u, v \in \mathbb{R}}?
13. Explain the concept of a tangent plane to a ruled surface S at a point P(u, v), denoted as T_{P}(S).
14. State the necessary and sufficient condition for a surface f(x, y, z) = 0 to be developable.
15. Define the metric tensor G of a surface S at a point P(u, v), denoted as G_{ij}(u, v).
16. Explain the first fundamental form of a surface S at a point P(u, v), denoted as I(u, v).
17. What are the second fundamental form and third fundamental form of a surface S at a point P(u, v), denoted as II(u, v) and III(u, v) respectively?
18. Define the Gaussian curvature K of a surface S at a point P(u, v), denoted as K(u, v).
19. What is an umbilic point P(u, v) on a surface S where the principal curvatures are equal, denoted as K_1(u, v) = K_2(u, v)?
20. Define the radius of curvature ρ of a normal section at an umbilic point P(u, v), denoted as $\rho(u, v)$.

21. Define the normal curvature k_n of a surface S at a point P(u, v), denoted as k_n(u, v).
22. State Meunier's theorem related to the normal curvature k_n of a surface S at a point P(u, v).
23. Define the lines of curvature on a surface S, denoted as L_1 and L_2.
24. What are the principal radii of a surface S at a point P(u, v), denoted as R_1(u, v) and R_2(u, v)?
25. State the relation between the first and second fundamental forms of a surface S at a point P(u, v).
26. Define the principal directions on a surface S at a point P(u, v), denoted as e_1(u, v) and e_2(u, v).
27. What are the principal curvatures k_1(u, v) and k_2(u, v) of a surface S at a point P(u, v)?
28. Define the mean curvature H of a surface S at a point P(u, v), denoted as H(u, v).
29. What is the first curvature k_1(u, v) of a surface S at a point P(u, v)?
30. Explain the concept of the Gaussian curvature K of a surface S at a point P(u, v).
31. State the relationship between the Gaussian curvature K(u, v) and the principal curvatures k_1(u, v) and k_2(u, v) of a surface S at a point P(u, v).
32. Define an umbilic point P(u, v) on a surface S where the principal curvatures are equal, denoted as k_1(u, v) = k_2(u, v).
33. What is the radius of curvature ρ(u, v) of a normal section through a point P(u, v) on a surface S represented by Z=F(x, y)?
34. Calculate the radius of curvature ρ(u, v) at a given section through any point of a surface S represented by Z=F(x, y).
35. Define asymptotic lines on a surface S, denoted as L_a1 and L_a2.
36. What are the fundamental magnitudes (L, M, N) of some important surfaces, where the first and second fundamental forms are given?
37. Define the concept of orthogonal trajectories on a surface S.
38. State the Existence and Uniqueness theorem for curves in 2D space.
39. Explain the characteristics of a Bertrand curve.
40. Define the involute of a curve in 2D space.
41. Explain the concept of the evolute of a curve in 2D space.
42. Define the concept of ruled surfaces.
43. What is a developable surface?
44. Explain the tangent plane condition for a ruled surface S at a point P(u, v).
45. State the necessary and sufficient condition for a surface f(x, y, z) = 0 to represent a developable surface.
46. Define the metric tensor G of a surface S at a point P(u, v).
47. Explain the first fundamental form I of a surface S at a point P(u, v).
48. What are the second and third fundamental forms II and III of a surface S at a point P(u, v)?
49. Define the Gaussian curvature K of a surface S at a point P(u, v).
50. State Meunier's theorem related to the normal curvature k_n of a surface S at a point P(u, v).

6/7 marks questions

1. Consider a curve C parametrized by a vector function r(t) = (x(t), y(t), z(t)) in 3D space. Derive the expressions for the tangent vector T(t), the principal normal vector N(t), and the binormal vector B(t) of the curve.
2. For a space curve C, the curvature κ(t) is given by κ(t) = ||dT/dt||, and the torsion τ(t) is given by τ(t) = (dT/dt) · (d^2r/dt^2) × (d^3r/dt^3). Calculate the curvature and torsion for the helix given by r(t) = (a cos(t), a sin(t), bt).
3. Prove that the Serret-Frenet formula for a curve C(t) = (x(t), y(t), z(t)) in 3D space is given by dT/dt = κN and dB/dt = -τN.
4. Given two curves C1(t) and C2(t) in 3D space, find their contact points, P, such that C1(t) and C2(t) have the same tangent and normal vectors at P.
5. A curve C(t) is said to be a Bertrand curve if it has at least two distinct osculating circles at every point. Prove that a curve is a Bertrand curve if and only if its curvature is constant.
6. Derive the equations for the involute and evolute of a plane curve C(x, y) in terms of its curvature κ(x, y) and the angle of the tangent θ(x, y) with the x-axis.
7. For a curve C parametrized by r(t) = (a cos(t), a sin(t), bt) in 3D space, find its osculating circle and osculating sphere at any point P(t).
8. Prove the Existence and Uniqueness theorem for space curves, stating that for any two points P and Q on a curve C, there exists a unique shortest path between P and Q lying entirely on C.
9. Consider the family of curves C(t) = (a cos(t), a sin(t), bt) in 3D space, where a and b are constants. Investigate the cases when these curves are geodesics (shortest paths) on a surface.
10. Prove that a surface S given by the equation z = f(x, y) represents a developable surface if and only if the Gaussian curvature K of S is zero everywhere.
11. Show that the tangent plane T_P(S) to a ruled surface S given by P(u, v) = C(u) + vN(u) at any point P(u, v) lies in a fixed direction in 3D space.
12. Prove that a surface S parametrized by P(u, v) = (x(u, v), y(u, v), z(u, v)) is developable if and only if there exists a pair of functions ϕ(u) and ψ(v) such that x(u, v) = a(u) cos(ϕ(u)), y(u, v) = a(u) sin(ϕ(u)), and z(u, v) = ψ(v) for some functions a(u) and ψ(v).
13. Show that a surface S given by the equation F(x, y, z) = 0 represents a developable surface if and only if the gradient vector ∇F is proportional to the normal vector N of S.
14. For a surface S parametrized by P(u, v) = (x(u, v), y(u, v), z(u, v)), derive the expression for the first fundamental form I(u, v) and its components g_{ij}(u, v).
15. Prove that the first fundamental form I(u, v) of a surface S is invariant under rigid motions (translations and rotations) in 3D space.
16. Derive the expressions for the second fundamental form II(u, v) and the third fundamental form III(u, v) of a surface S parametrized by P(u, v) = (x(u, v), y(u, v), z(u, v)).

17. For a surface S, show that the Gaussian curvature K is given by $K = (LN - M^2) / (EG - F^2)$, where E, F, and G are components of the first fundamental form, and L, M, and N are components of the second fundamental form.
18. Consider a surface S with constant Gaussian curvature K. Prove that the mean curvature H of S is given by $H = (E + G)K/2$.
19. Show that for a surface S with constant Gaussian curvature K, the principal curvatures k_1 and k_2 are constant and satisfy the relation $k_1k_2 = K$.
20. Derive the expression for the normal curvature $k_n(u, v)$ of a surface S parametrized by P(u, v) in the direction of the normal vector N(u, v).
21. Prove Meunier's theorem, which states that if a surface S has zero Gaussian curvature K at a point P, then S has zero normal curvature in every direction at P.
22. Show that the lines of curvature on a surface S are the orthogonal trajectories of the lines of curvature on S.
23. Derive the formula for the principal radii of curvature R_1 and R_2 of a surface S at a point P(u, v) in terms of the first and second fundamental forms.
24. Consider a surface S parametrized by P(u, v) = (x(u, v), y(u, v), z(u, v)), and let E, F, and G be components of the first fundamental form, and L, M, and N be components of the second fundamental form. Prove that the normal curvature k_n is given by $k_n = (LN - M^2) / (EG - F^2)$.
25. Prove that for a surface S with constant Gaussian curvature K, the lines of curvature coincide with the lines of symmetry of S.
26. Derive the expressions for the principal directions e_1 and e_2 and the principal curvatures k_1 and k_2 of a surface S parametrized by P(u, v).
27. Show that for a surface S with constant Gaussian curvature K, the principal curvatures k_1 and k_2 are constant and satisfy the relation $k_1k_2 = K$.
28. Consider a surface S with constant mean curvature H. Prove that the Gaussian curvature K of S is given by $K = H^2$.
29. Derive the formula for the mean curvature H of a surface S at a point P(u, v) in terms of the first and second fundamental forms.
30. Show that the Gaussian curvature K and the mean curvature H are invariant under isometries (conformal transformations) of the surface S.