MULTIVARIABLE CALCULUS

CORE PAPER - XI

1 mark questions

1. What is partial differentiation?
2. Define total differentiability and differentiability.
3. State the chain rule for two independent parameters.
4. What is the tangent plane?
5. State the sufficient condition for differentiability.
6. Define Lagrange multipliers.
7. What are constrained optimization problems?
8. Define vector field.
9. What is the maximal property of the gradient?
10. Define double integration over nonrectangular region.
11. Define triple integrals.
12. What is a parallelepiped?
13. Define cylindrical and spherical coordinates.
14. What is the volume by triple integrals?
15. What is change of variables in double and triple integrals?
16. Define line integrals.
17. What are the applications of line integrals?
18. State the fundamental theorem for line integrals.
19. What are conservative vector fields?
20. What is the independence of path?
21. The functions of several variables include ________ and ________.
22. The limit and continuity of functions of two variables are important concepts in ________.
23. Partial differentiation is used to find the rate of change of a function with respect to ________ variable(s) while holding other variables constant.
24. Total differentiability and differentiability are conditions that determine if a function is ________ at a point.
25. The chain rule allows us to find the derivative of a composite function with ________ independent parameters.
26. Directional derivatives measure the rate of change of a function in the direction of a given ________.
27. The gradient is a vector that points in the direction of maximum ________ and has the property of being normal to level curves or surfaces.
28. Tangent planes are used to approximate functions near a given ________ point.
29. Extrema of functions of two variables can be found using methods such as the ________ multipliers or constrained optimization problems.
30. Vector fields have properties such as divergence and curl, which describe their behavior in terms of ________ and rotation, respectively.
31. Find the gradient of the function $f(x,y) = 3x^2 + 4y^3$ at the point (1,2).
32. Determine if the function $g(x,y) = x^3 + 2xy^2 - y^3$ is differentiable at the point (0,0).

33. Find the equation of the tangent plane to the surface $z = x^2 + y^2 - 4$ at the point (1,1,2).
34. Use the chain rule to find the derivative of $f(x,y) = \sin(x^2 + y^2)$ with respect to x.
35. Compute the directional derivative of $h(x,y) = e^{(x+y)}$ in the direction of the vector <1,2> at the point (0,0).
36. Evaluate the limit lim(x,y)->(0,0) $(x^2 + y^2)/(x+y)$.
37. Determine if the function $k(x,y) = 1/x + y$ is continuous at the point (1,2).
38. Find the partial derivatives of the function $p(x,y) = x^3 + 2xy - y^2$ with respect to x and y.
39. Show that the function $q(x,y) = x^2 + y^2$ is totally differentiable at the point (1,1).
40. Give an example of a function of two variables that takes several input variables and returns a single output value.
41. Use Lagrange multipliers to find the maximum and minimum values of $f(x,y) = x^2 + 4y^2$ subject to the constraint $x + 2y = 5$.
42. Solve the constrained optimization problem of maximizing $g(x,y) = xy$ subject to the constraint $x^2 + y^2 = 1$.
43. Find the divergence and curl of the vector field F(x,y) = <2x-y, 3x+2y>.
44. Evaluate the double integral ∬R $(x^2 + y^2)$ dA, where R is the region bounded by the curves $y = x$ and $y = x^2$.
45. Use polar coordinates to evaluate the double integral ∬R xy dA, where R is the region enclosed by the cardioid $r = 1 + \cos(\theta)$.
46. Compute the double integral ∬R $e^{(x2+y2)}$ dA, where R is the region bounded by the curves $x = 0$, $y = 0$, and $x^2 + y^2 = 1$.
47. Find the gradient of the function $f(x,y,z) = x^2 + y^2 + z^2$ at the point (1,1,1).
48. Use the chain rule for two independent parameters to find the derivative of g(x,y) = $\sin(x^{2y})$ with respect to y.
49. Compute the directional derivative of $h(x,y,z) = xyz$ in the direction of the vector <1,1,1> at the point (2,3,4).
50. Give an example of a vector field that assigns a vector to each point in space.
51. Use the unique factorization theorem in Z[x] to factor the polynomial $f(x) = x^3 - 3x^2 + 3x - 1$.
52. Show that the element 2 + sqrt(3) in Q(sqrt(3)) is irreducible.
53. Find a prime element in Z[i] that generates a prime ideal.
54. Determine if Z[sqrt(-5)] is a unique factorization domain.
55. Use a polynomial ring over Z to define the expression (3x^2 - 2x + 1)(x^3 + 2x).
56. Use the division algorithm for polynomials to divide f(x) = x^3 - 3x^2 + 3x - 1 by g(x) = x-1.
57. Show that Z[x] is a Euclidean domain with respect to the Euclidean function f(x) = degree of polynomial.
58. Use a Euclidean element in Z[x] to perform division with remainder using the Euclidean function.
59. Find the principal ideal generated by the element 2 in Z.
60. Show that the element 5 in Z is a prime element that generates a prime ideal.

2 marks questions

1. Define the maximal and normal property of the gradient.
2. State the sufficient condition for differentiability of a function of two variables.
3. Define the tangent plane to a surface.
4. What is the chain rule for one independent parameter?
5. Define the directional derivative of a function of two variables.
6. State the definition of limit of a function of two variables.
7. Define the continuity of a function of two variables.
8. What is partial differentiation?
9. Define total differentiability and differentiability.
10. State the definition of a function of several variables.
11. Define the method of Lagrange multipliers.
12. What are constrained optimization problems?
13. State the definition of a vector field.
14. Define divergence and curl.
15. What are double integrals over non-rectangular regions?
16. Define double integrals in polar coordinates.
17. State the definition of double integration over rectangular region.
18. What is the maximal and minimal property of the gradient?
19. What is the chain rule for two independent parameters?
20. Define directional derivatives.
21. What is the unique factorization theorem in Z[x]?
22. Define an irreducible element in an integral domain.
23. State the definition of a prime element in an integral domain.
24. What is a unique factorization domain?
25. Define a polynomial ring over a commutative ring.
26. What is the division algorithm for polynomials?
27. Define a principal ideal domain.
28. What is the Eisenstein criterion?
29. What is the irreducibility test for polynomials?
30. What is the generator theorem for principal ideal domains?
31. Define divisibility in an integral domain.
32. What is an irreducible element in an integral domain?
33. State the definition of a factorial domain.
34. What is a unique factorization domain?
35. Define the Euclidean function in an integral domain.
36. What is a Euclidean domain?
37. Define a Euclidean element in an integral domain.
38. What is the principal ideal of an element in an integral domain?
39. What is the generator theorem for principal ideal domains?
40. State the definition of a prime element in an integral domain.

41. Determine if 15 divides 105 in Z.
42. Show that the element 2 + sqrt(3) in Q(sqrt(3)) is irreducible.
43. Define a factorial domain and give an example.

44. Show that Z[sqrt(-5)] is a unique factorization domain.
45. Use the Euclidean function f(x) = absolute value of x to find gcd(24,36) in Z.
46. Define a Euclidean domain and give an example.
47. Use a Euclidean element in Z to perform division with remainder using the Euclidean function.
48. Find the principal ideal generated by the element 2 in Z.
49. State the generator theorem for principal ideal domains.
50. Show that the element 5 in Z is a prime element that generates a prime ideal.

6/7 marks questions

1. Define the gradient of a function of two variables and explain its maximal and normal property.
2. State and prove the sufficient condition for differentiability of a function of two variables.
3. Define the tangent plane to a surface at a point and derive an equation for it.
4. Explain the chain rule for one independent parameter and give an example.
5. Define the directional derivative of a function of two variables and compute it for a given function.
6. Define the limit of a function of two variables and prove its uniqueness.
7. Define the continuity of a function of two variables and give an example of a function that is continuous but not differentiable.
8. Explain partial differentiation and derive the formula for it.
9. Define total differentiability and differentiability and give an example of a function that is differentiable but not totally differentiable.
10. Define a function of several variables and give an example.
11. Explain the method of Lagrange multipliers and use it to solve a constrained optimization problem.
12. Define constrained optimization problems and give an example.
13. Define a vector field and give an example.
14. Define divergence and curl of a vector field and derive their formulas.
15. Explain how to evaluate double integrals over non-rectangular regions and give an example.
16. Define double integrals in polar coordinates and derive the formula for it.
17. State the definition of double integration over rectangular region and derive the formula for it.
18. Explain the maximal and minimal property of the gradient and give an example.
19. Explain the chain rule for two independent parameters and give an example.
20. Define directional derivatives and compute them for a given function.
21. Define the unique factorization theorem in Z[x] and prove it.
22. Define an irreducible element in an integral domain and give an example.
23. State the definition of a prime element in an integral domain and give an example.
24. Define a unique factorization domain and give an example.
25. Define a polynomial ring over a commutative ring and give an example.
26. Explain the division algorithm for polynomials and use it to find the quotient and

remainder of a given polynomial.
27. Define a principal ideal domain and give an example.
28. Explain the Eisenstein criterion for irreducibility of polynomials and use it to prove that a given polynomial is irreducible.
29. Explain the irreducibility test for polynomials and use it to prove that a given polynomial is irreducible.
30. State and prove the generator theorem for principal ideal domains.
31. Define divisibility in an integral domain and give an example.
32. Define an irreducible element in an integral domain and give an example.
33. State the definition of a factorial domain and give an example.
34. Define a unique factorization domain and give an example.
35. Define the Euclidean function in an integral domain and give an example.
36. Explain what a Euclidean domain is and give an example.
37. Define a Euclidean element in an integral domain and give an example.
38. Explain what the principal ideal of an element in an integral domain is and give an example.
39. State the generator theorem for principal ideal domains and prove it.
40. Define a prime element in an integral domain and give an example.