THEORY OF REAL FUNCTIONS

CORE PAPER- V

1 mark questions

1. What is L'Hospital's rule used for?
2. Define an indeterminate form.
3. State Cauchy's Mean Value Theorem.
4. What is Taylor's theorem with Lagrange's form of remainder?
5. What is Taylor's theorem with Cauchy's form of remainder?
6. How is Taylor's theorem used with convex functions?
7. Define a relative extremum.
8. What is the main idea behind Taylor's series?
9. Define Maclaurin's series.
10. Write down the Taylor series expansion for the exponential function.
11. Define Riemann integration.
12. What are the inequalities of upper and lower sums?
13. What are the Riemann conditions of integrability?
14. Define the Riemann sum.
15. Are continuous functions always Riemann integrable? Why or why not?
16. What is the Intermediate Value Theorem for Integrals?
17. What is the Fundamental Theorem of Calculus?
18. Are monotone functions always Riemann integrable? Why or why not?
19. What is a piecewise continuous function?
20. What are the properties of the Riemann integral?
21. Define an improper integral.
22. State the convergence conditions for Beta and Gamma functions.
23. Define pointwise convergence.
24. Define uniform convergence.
25. What are the theorems on continuity of a limit function?
26. What are the theorems on differentiability of a limit function?
27. What are the theorems on integrability of a limit function?
28. How is uniform convergence different from pointwise convergence?
29. How does uniform convergence impact the properties of a sequence of functions?
30. Define a series of functions.
31. What is the Cauchy criterion for uniform convergence?
32. Define the Weierstrass M-Test.
33. What is limit superior?
34. What is limit inferior?
35. What is a power series?
36. What is the radius of convergence of a power series?
37. State the Cauchy-Hadamard Theorem.
38. Can a power series be differentiated term by term? Why or why not?
39. Can a power series be integrated term by term? Why or why not?

40. State Abel's Theorem.
41. State the Weierstrass Approximation Theorem.
42. What is the convergence criterion for a series of functions?
43. How can the Weierstrass M-test be used to determine the convergence of a series of functions?
44. How does Abel's theorem relate to power series?
45. How does the Weierstrass Approximation Theorem apply to continuous functions?
46. What conditions must be satisfied for a function to have a Taylor series representation?
47. What is the difference between an infinite series and a power series?
48. How does the radius of convergence of a power series impact its behavior?
49. How does the limit superior of a sequence differ from its limit?
50. How does the limit inferior of a sequence differ from its limit?
51. L'Hospital's rule is used to solve __________ forms.
52. An indeterminate form is an expression involving two functions whose limit cannot be determined from the __________ of the individual limits.
53. Cauchy's Mean Value Theorem generalizes the concept of the __________ Mean Value Theorem.
54. In Taylor's theorem with Lagrange's form of remainder, the remainder is expressed as a __________ integral.
55. Taylor's theorem with Cauchy's form of remainder uses the __________ of the function.
56. Taylor's theorem for convex functions helps to find the __________ of the function.
57. In calculus, a relative extremum is also known as a __________.
58. The Taylor series provides an approximation of a function near a point called the __________.
59. The Maclaurin series is a Taylor series expansion of a function about __________.
60. The Taylor series expansion for the exponential function is a __________ series.
61. In Riemann integration, the integral of a function is calculated as the __________ of rectangles under the graph of the function.
62. The inequalities of upper and lower sums relate to the __________ of the integral.
63. The Riemann conditions of integrability ensure that the function is __________ over the interval.
64. The Riemann sum is an approximation for the __________ of a function over an interval.
65. Continuous functions are always Riemann __________.
66. The Intermediate Value Theorem for Integrals states that if a function is __________ on an interval, then it takes on every value between the least upper bound and greatest lower bound of its range.
67. The Fundamental Theorem of Calculus connects differentiation and __________.
68. Monotone functions are always Riemann __________.
69. A piecewise continuous function is a function that is continuous on its __________.

70. The properties of the Riemann integral are used to calculate the __________ of functions.
71. An improper integral is an integral with an __________ limit or integrand.
72. The convergence conditions for Beta and Gamma functions are based on the __________ of the functions.
73. Pointwise convergence refers to the convergence of a sequence of functions at each __________ independently.
74. Uniform convergence refers to the convergence of a sequence of functions at all __________ simultaneously.
75. The theorems on continuity of a limit function state that the limit of a sequence of continuous functions is a __________ function.
76. The theorems on differentiability of a limit function state that the limit of a sequence of differentiable functions is a __________ function.
77. The theorems on integrability of a limit function state that the limit of a sequence of integrable functions is an __________ function.
78. Uniform convergence differs from pointwise convergence in terms of the __________ of convergence.
79. Uniform convergence impacts the __________, __________, and __________ of a sequence of functions.
80. A series of functions is a __________ of functions.
81. The Cauchy criterion for uniform convergence states that a series of functions converges uniformly if and only if for every __________, there is a positive integer N such that the absolute difference between the sums of the first m and n terms is less than __________ for all m and n greater than N.
82. The Weierstrass M-Test is a __________ test for uniform convergence.
83. Limit superior is the __________ limit of a sequence.
84. Limit inferior is the __________ limit of a sequence.
85. A power series is a series of the form __________.
86. The radius of convergence of a power series is the radius of the largest __________ in which the series converges.
87. The Cauchy-Hadamard theorem gives the __________ of convergence of a power series.
88. A power series can be differentiated term by term within its __________ of convergence.
89. A power series can be integrated term by term within its __________ of convergence.
90. Abel's theorem concerns the __________ of convergence of power series.
91. The Weierstrass Approximation theorem states that every continuous function defined on a closed interval can be uniformly approximated as closely as desired by a __________.

2/3 marks questions

1. In L'Hospital's rule, the indeterminate form 0/0 or ∞/∞ is often encountered. How is this rule used to resolve such forms?

2. Cauchy's mean value theorem is an extension of the mean value theorem. It states that for two functions f and g, which are continuous on the closed interval [a,b] and differentiable on the open interval (a,b), there exists some c in (a,b), such that (f(b)-f(a))(g'(c)) = (g(b)-g(a))(f'(c)). Give an example illustrating this theorem.
3. Taylor's theorem with Lagrange's form of remainder states that if a function f and its n first derivatives are continuous in a closed interval [a,b] that contains the number x, and the (n+1)th derivative is existent on the open interval (a,b), then the function satisfies the following equation: $f(x) = P_n(x) + R_n(x)$. What do $P_n(x)$ and $R_n(x)$ represent?
4. Explain how Taylor's theorem can be applied to convex functions. Give an example illustrating this application.
5. Explain the concept of Riemann sums and how it leads to the definition of Riemann integrals.
6. In the context of Riemann integration, what do we mean by upper and lower sums? Give a mathematical definition.
7. State and explain the conditions for Riemann integrability.
8. What does the Intermediate Value Theorem for Integrals state mathematically?
9. The Fundamental Theorem of Calculus connects the concept of an integral to the concept of a derivative. What does this theorem state mathematically?
10. Define an improper integral and give an example.
11. The convergence conditions for Beta and Gamma functions involve the concepts of continuity, differentiability, and integrability. Explain these conditions mathematically.
12. Explain the concept of pointwise and uniform convergence for a sequence of functions with appropriate mathematical notations.
13. State and explain the theorems regarding the continuity, derivability, and integrability of the limit function of a sequence of functions.
14. Define a series of functions and give an example.
15. Explain the Cauchy criterion for uniform convergence. Give an example illustrating this criterion.
16. The Weierstrass M-Test is a method to prove uniform convergence of a series of functions. What does this test state mathematically?
17. The term 'limit superior' refers to the least upper bound of the set of limit points of a sequence. Define it mathematically.
18. The term 'limit inferior' refers to the greatest lower bound of the set of limit points of a sequence. Define it mathematically.
19. State and explain the Cauchy-Hadamard theorem for the radius of convergence of a power series.
20. State and explain Abel's theorem and the Weierstrass Approximation Theorem.

6/7 marks questions

1. Discuss L'Hospital's Rules and its significance in resolving indeterminate forms. Provide a detailed example illustrating its use.

2. Explain Cauchy's Mean Value Theorem. How does it generalize the concept of the Mean Value Theorem? Give a specific mathematical example to illustrate this.
3. Describe the application of Taylor's Theorem with Lagrange's form of remainder. Provide a thorough example with a function of your choice.
4. Discuss Taylor's theorem with Cauchy's form of remainder and illustrate its application with a relevant example.
5. Discuss the application of Taylor's Theorem to convex functions. Give a detailed example to illustrate your explanation.
6. Elaborate on what relative extrema are in calculus. Provide a thorough example to explain your point.
7. Discuss in detail the concept of Taylor's series and Maclaurin's series. Provide a thorough example to illustrate your explanation.
8. Discuss the significance of the expansions of exponential and trigonometric functions in Taylor's series and Maclaurin's series.
9. Discuss the concept of Riemann integration. How do the inequalities of upper and lower sums come into play?
10. Explain the concept of Riemann sums and how they lead to the definition of Riemann integrals.
11. What are the conditions for a function to be Riemann integrable? Provide a detailed explanation.
12. Discuss the properties of the Riemann integral and how they enable the calculation of the area under curves.
13. Discuss in detail the concept of Intermediate Value theorem for Integrals.
14. Explain the fundamental theorems of Calculus and their importance in the study of mathematics.
15. Discuss the convergence of Beta and Gamma functions. Provide a detailed explanation and examples for each.
16. Discuss the concept of pointwise and uniform convergence of a sequence of functions.
17. Explain how the theorems on continuity, derivability, and integrability of the limit function of a sequence of functions relate to each other.
18. Discuss the concept of improper integrals and how they extend the concept of definite integrals. Provide an example to illustrate your explanation.
19. Discuss in detail the concept of a series of functions.
20. Explain the Cauchy criterion for uniform convergence and the Weierstrass M-Test.
21. Discuss the concepts of limit superior and limit inferior of a sequence.
22. Discuss the concept of a power series, including the radius of convergence. Provide a detailed example.
23. Discuss and provide examples of the differentiation and integration of power series.
24. Explain the significance of Abel's Theorem and Weierstrass Approximation Theorem in the context of power series. Provide a detailed example.