DIFFERTIAL GEOMETRY
DSE-III

1 mark questions

1. Define a space curve.
2. Differentiate between space curves and planar curves.
3. Write the Serret-Frenet formulas for a space curve.
4. Define curvature of a space curve.
5. Define torsion of a space curve.
6. Write the equation for the osculating circle of a space curve.
7. Explain the concept of osculating circles and spheres.
8. State the condition for the existence of a space curve.
9. Define a space curve in three-dimensional space.
10. Differentiate between space curves and planar curves using mathematical notation.
11. Write the expression for the curvature (κ) of a space curve in terms of derivatives of its position vector.
12. Define torsion (τ) of a space curve and write its formula.
13. State the Serret-Frenet formulas for a space curve.
14. Provide the equation for the osculating circle of a space curve.
15. Write the equation for the osculating sphere of a space curve.
16. State the condition that ensures the existence of a space curve.
17. Define evolutes and involutes of curves.
18. Explain the concept of parametric curves on surfaces.
19. Define surfaces of revolution.
20. Define helicoids and provide an example.
21. Explain the concept of direction coefficients.
22. State the first fundamental form for a surface.
23. State the second fundamental form for a surface.
24. Define evolutes of curves and write the mathematical expression for the evolute.
25. Explain the concept of involutes of curves using mathematical notation.
26. Define parametric curves on surfaces and give an example.
27. Write the equation for a surface of revolution.
28. Define a helicoid and express its parametric equations.
29. Explain the concept of direction coefficients for a curve on a surface.
30. Write the formula for the first fundamental form of a surface.
31. Provide the mathematical expression for the second fundamental form of a surface.
32. Define principal curvature of a surface.
33. Define Gaussian curvature of a surface.
34. Explain the concept of lines of curvature.
35. State Euler's theorem related to lines of curvature.
36. Write Rodrigues' formula.
37. Define conjugate lines on a surface.
38. Define asymptotic lines on a surface.
39. Explain the minimal surfaces concept.
40. Define principal curvature (k1, k2) of a surface and write its formula.

41. Write the expression for the Gaussian curvature (K) of a surface using the principal curvatures.
42. Explain the concept of lines of curvature on a surface using mathematical notation.
43. State Euler's theorem for lines of curvature and write the corresponding equation.
44. Write Rodrigues' formula for the angle between the normal to a surface and the axis of rotation.
45. Define conjugate lines on a surface and express them mathematically.
46. Define asymptotic lines on a surface and provide their mathematical definition.
47. Explain the concept of minimal surfaces using mathematical notation.
48. Define geodesics on a surface.
49. Write the canonical geodesic equations.
50. Explain the nature of geodesics on a surface of revolution.
51. State Clairaut's theorem for geodesics.
52. Define normal property of geodesics.
53. Define geodesic curvature.
54. State the Gauss-Bonnet theorem.
55. Define surfaces of constant curvature.
56. Define geodesics on a surface and explain their importance using mathematical notation.
57. Write the canonical geodesic equations for a surface in terms of Christoffel symbols.
58. Explain the nature of geodesics on a surface of revolution using mathematical expressions.
59. State Clairaut's theorem for geodesics and provide its mathematical formulation.
60. Define the normal property of geodesics and write its mathematical representation.
61. Define geodesic curvature and express it using mathematical notation.
62. State the Gauss-Bonnet theorem for surfaces and write its mathematical form.
63. Define surfaces of constant curvature and provide a mathematical example.

2/3 marks questions

1. Explain the concept of a space curve using a parametric equation: $r(t) = \langle x(t), y(t), z(t)\rangle$.
2. Differentiate between space curves and planar curves using examples.
3. Derive the formula for curvature (κ) of a space curve in terms of its derivatives.
4. Define torsion (τ) of a space curve using the cross product and provide its formula.
5. Write the Serret-Frenet formulas for a space curve in terms of tangent, principal normal, and binormal vectors.
6. Calculate the curvature and torsion of a given space curve using the Serret-Frenet formulas.
7. Explain how to determine the osculating circle of a space curve using its curvature.
8. Find the center and radius of the osculating circle of a space curve at a specific point.
9. Discuss the existence of space curves by analyzing curvature and torsion.
10. Calculate the curvature and torsion for a helix given its parametric equations.

11. Define evolutes and involutes of curves and explain their relationship.
12. Find the evolute of a given parametric curve on a plane.
13. Explain how parametric curves on surfaces are defined and give an example.
14. Derive the parametric equations for a surface of revolution using a curve in the plane.
15. Provide the parametric equations for a helicoid and explain its geometric properties.
16. Compute the direction coefficients of a tangent vector to a curve on a surface.
17. Write the first fundamental form for a surface in terms of its parametric equations.
18. Calculate the area of a small patch on a surface using the first fundamental form.
19. Define the second fundamental form for a surface and explain its significance.
20. Compute the Gaussian curvature (K) of a surface using the first and second fundamental forms.
21. State the definitions of principal curvatures (k1, k2) and explain their geometric interpretation.
22. Calculate the Gaussian curvature (K) of a surface using its principal curvatures.
23. Define lines of curvature on a surface and explain their orientation with respect to the principal directions.
24. Apply Euler's theorem to find the relationship between the principal curvatures and Gaussian curvature.
25. Use Rodrigues' formula to determine the angle between the normal to a surface and a fixed direction.
26. Define conjugate lines on a surface and explain their orthogonal relationship with lines of curvature.
27. Describe asymptotic lines on a surface and explain their geometric behavior.
28. Derive the formula for the mean curvature (H) of a surface in terms of its principal curvatures.
29. Explain the concept of minimal surfaces and provide an example.
30. Define geodesics on a surface and explain their connection to shortest paths.
31. Write the canonical geodesic equations for a surface in terms of Christoffel symbols.
32. Describe the nature of geodesics on a surface of revolution using mathematical expressions.
33. Apply Clairaut's theorem to find geodesics on a surface of revolution.
34. Explain the normal property of geodesics and its relation to the curvature of the surface.
35. Define geodesic curvature and explain its role in measuring deviation from a straight path.
36. State the Gauss-Bonnet theorem and explain its significance for surfaces.
37. Define surfaces of constant curvature and provide examples.
38. Determine the curvature of a given surface and classify it as a surface of constant curvature.
39. Explain the normal property of geodesics and its relation to the curvature of the surface.
40. Define geodesic curvature and its significance in measuring deviation from a straight path.

41. State the Gauss-Bonnet theorem and explain its connection between curvature and topology.
42. Define surfaces of constant curvature and provide examples.
43. Determine the curvature of a given surface and classify it as a surface of constant curvature.

<u>6/7 marks questions</u>

1. Prove the Serret-Frenet formulas for a space curve and demonstrate how they provide an orthogonal basis.
2. Explain the geometric properties of the osculating circle of a space curve and derive its equation.
3. Prove the existence of space curves by showing that continuous functions of the Serret-Frenet frame yield valid curves.
4. Compare and contrast space curves and planar curves in terms of their dimensionality and mathematical representation.
5. Calculate the curvature and torsion of a given space curve using the Serret-Frenet formulas and provide a practical example.
6. Explain how to determine the center and radius of the osculating circle of a space curve using its curvature. Derive the equation for the osculating circle.
7. Discuss the properties of osculating circles and spheres of a space curve and their relationships with the curve's behavior.
8. Prove the existence of space curves by considering continuity and differentiability conditions, and use Frenet-Serret formulas to ensure well-defined curves.
9. Derive the evolute of a parametric curve in the plane and explain the concept of evolutes.
10. Explain the construction of involutes of a given curve and illustrate with an example.
11. Explain the geometric properties of a helicoid and discuss how it is a non-developable surface.
12. Define evolutes of curves and explain their relationship with the original curve. Derive the equation of the evolute.
13. Explain the concept of involutes of curves, providing a mathematical definition and a real-world example.
14. Define parametric curves on surfaces and illustrate their representation using surface parameters. Give an example.
15. Calculate the area of a small patch on a surface using the first fundamental form and integration. Interpret the result in terms of surface area.
16. Derive the equation for lines of curvature on a surface in terms of the coefficients of the first and second fundamental forms.
17. Explain how to determine conjugate lines on a surface and derive their equation.
18. Discuss the properties of minimal surfaces and provide examples of naturally occurring minimal surfaces.
19. Explain the concept of lines of curvature on a surface and their connection to the principal directions. Describe how lines of curvature can be obtained.

20. Use Rodrigues' formula to determine the angle between the normal to a surface and a fixed direction. Discuss the implications of this formula on surface orientation.
21. Define conjugate lines on a surface and explain their orthogonality to lines of curvature. Provide a mathematical example.
22. Describe asymptotic lines on a surface and explain their geometric properties. Analyze the relationship between principal curvatures and asymptotic directions.
23. Explain the concept of minimal surfaces and provide a real-world example. Discuss the importance of minimal surfaces in various applications.
24. Derive the canonical geodesic equations for a surface using the Christoffel symbols and explain their significance.
25. Discuss the nature of geodesics on a surface of revolution and use Clairaut's theorem to find geodesics on specific surfaces.
26. Prove the Gauss-Bonnet theorem, illustrating the relationship between curvature and topology for closed surfaces.
27. Explain the concept of surfaces of constant curvature and provide examples of positively curved, negatively curved, and flat surfaces.
28. Compare and contrast geodesics, lines of curvature, and asymptotic lines on a surface, highlighting their distinct geometric properties.
29. Describe the nature of geodesics on a surface of revolution using mathematical expressions. Discuss how the surface's symmetry affects geodesic behavior.
30. Explain the normal property of geodesics and its relation to the curvature of the surface. Illustrate how the normal property influences geodesic behavior.
31. Define geodesic curvature and explain its role in measuring deviation from a straight path. Relate geodesic curvature to the concept of torsion.
32. State the Gauss-Bonnet theorem and explain its connection between curvature and topology. Discuss its implications for surfaces and their geometric properties.
33. Define surfaces of constant curvature and provide examples of surfaces with positive, negative, and zero constant curvature. Analyze their distinctive features.
34. Determine the curvature of a given surface and classify it as a surface of constant curvature. Explain the geometrical characteristics of surfaces with different constant curvatures.