P – 303

STOCHASTIC PROCESS

PG 3rd SEMESTER

Short Type Questions

1. Define a stochastic process.
2. What is a stationary process?
3. Explain the concept of martingales.
4. What is a random walk in the context of stochastic processes?
5. Define a ruin problem in the context of stochastic processes.
6. How is the expected duration of a game calculated in stochastic processes?
7. What is the generating function of the duration of a game?
8. Explain the concept of a generating function for first passage times.
9. What is a random walk in the plane?
10. What is a random walk in space?
11. How are states classified in Markov chains?
12. What is the stability of a Markov system?
13. Explain the concept of a finite irreducible chain in Markov chains.
14. What is the ergodic theorem in the context of Markov chains?
15. Describe the graph theoretic approach to Markov chains.
16. What are reducible chains in Markov theory?
17. What is the ergodic theorem for reducible chains?
18. How does a Markov chain with continuous state space differ from a discrete one?
19. What are non-homogeneous Markov chains?
20. Explain the concept of higher transition probabilities in Markov chains.
21. Define a Poisson process.
22. What are some properties of a Poisson process?
23. How is a Poisson process related to other probability distributions?
24. What is a pure birth process in Markov theory?
25. Explain the Yule-Furry process.
26. Describe the birth-immigration process.
27. What is a time-dependent Poisson process?
28. Define a pure death process.
29. Explain the concept of birth and death processes in Markov theory.
30. What are the Chapman-Kolmogorov forward and backward equations?
31. What is Brownian motion in the context of stochastic processes?
32. Define a Wiener process.
33. What are the differential equations associated with a Wiener process?

34. Explain the concept of Kolmogorov equations in continuous state space.
35. How is the first passage time distribution calculated for a Wiener process?

Long Question

1. Explain the fundamental concepts of a stochastic process, including the sample space, time index, and random variables associated with it. Provide examples to illustrate different types of stochastic processes.
2. Explain Gambler's ruin Problem in Stochastic Process.
3. Explain One dimensional Random Walk in Stochastic Process.
4. Explain Two dimensional Random Walk in Stochastic Process.
5. Explain the concept of state classification in Markov chains. Discuss the criteria used to classify states as transient, recurrent, and absorbing. Provide examples to illustrate each type of state.
6. The t.p.m. of a Markov Chain $\{X_n , n = 1, 2 , \dots .\}$ having three states 1, 2 and 3 is

$$\begin{pmatrix} 0.1 & 0.5 & 0.4 \\ 0.6 & 0.2 & 0.2 \\ 0.3 & 0.4 & 0.3 \end{pmatrix}$$

   And initial distribution is $\Pi_0 = (0.7, 0.2, 0.3)$
   Find (i) $\Pr\{X_2 = 3\}$
   (ii) $\Pr\{X_3 = 3, X_2 = 2, X_1 = 3, X_0 = 1\}$
7. Describe the procedure for determining higher-order transition probabilities in a Markov chain. Use a specific example to calculate and interpret these probabilities.
8. Prove that, State j is Persistent $iff\ \sum_{n=0}^{\infty} P_{jj}^{(n)} = \infty$
9. The t.p.m. of a Markov Chain $\{X_n, n = 1,2, \dots .\}$ having states 1,2, 3 and 4 is

$$P = \begin{bmatrix} 1/3 & 1/3 & 1/3 & 0 \\ 1/4 & 0 & 3/4 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 2/3 & 1/3 \end{bmatrix}$$

   Then find which states are transient or persistent.
10. Define irreducible Markov chains and explain their significance. Discuss the concept of limiting behavior in the context of finite irreducible chains. Provide examples to illustrate different types of limiting behavior.
11. Explain the Ergodic Theorem in Markov chains. Discuss its implications and limitations. Provide a real-world application where the Ergodic Theorem is used for analysis.

12. Describe the graph theoretic approach to analyzing Markov chains. Explain how the transition matrix is related to the adjacency matrix of the associated graph. Provide an example to demonstrate this approach.

13. Consider a Markov chain with the following transition matrix:

$$\begin{pmatrix} .6 & .4 & 0 \\ .3 & .4 & .3 \\ .1 & .2 & .7 \end{pmatrix}$$

i. Classify the states in this Markov chain as transient, recurrent, or absorbing.
ii. Calculate the limiting probabilities for each state.
iii. Determine the expected number of steps it takes for the Markov chain to reach state 3, starting from state 1.
iv. Using the graph theoretic approach, draw the directed graph associated with this Markov chain.
v. Identify the closed classes within the chain and calculate the expected return time for each closed class.

14. Explain the Poisson process in detail, including its properties such as independence and memory lessness. Provide examples of real-world phenomena that can be modeled using a Poisson process.
15. Define a pure birth process and discuss its characteristics.
16. Explain the Yule-Furry process, including its birth and death rates. Discuss its applications and differences from the standard Poisson process.
17. Describe the birth-immigration process and its components. Discuss scenarios where the birth-immigration process is a suitable model and how it differs from other processes.
18. Explain the concept of time-dependent Poisson processes. Discuss how the intensity function can vary over time and its impact on the process. Provide examples of practical situations where time-dependent Poisson processes are useful.
19. Define a pure death process and discuss its characteristics. Provide examples to illustrate when and how a pure death process can be applied in modeling.
20. Discuss birth and death processes in detail. Explain the role of birth and death rates, as well as their implications for the long-term behavior of the process. Provide a numerical example to demonstrate the calculations.
21. Derive the Chapman-Kolmogorov forward equations for a discrete-time Markov chain. Explain the significance of these equations and how they are used to predict future state probabilities.
22. Derive the Chapman-Kolmogorov backward equations for a discrete-time Markov chain. Discuss the differences between the forward and backward equations and provide an example where these equations are applied to solve a specific problem.
23. Define Brownian motion and explain its key properties, including the Gaussian nature of increments and the concept of self-similarity. Provide examples of natural phenomena that can be modeled using Brownian motion.
24. Discuss the Wiener process (also known as the standard Brownian motion) in detail, including its mathematical definition and properties. Explain why it's considered a fundamental continuous-time stochastic process.

25. Derive the stochastic differential equations (SDEs) that govern the behavior of a Wiener process. Discuss how these equations relate to the concept of drift and diffusion in the context of stochastic calculus.
26. Explain the Kolmogorov equations in the context of continuous-time Markov processes. Discuss their importance in describing the evolution of probability densities. Provide an example of solving the Kolmogorov equations for a specific Wiener process.