GOVERNMENT AUTONOMOUS COLLEGE, PANPOSE
ROURKELA
Subject –Statistics (UG-1st year, 1st Semester) Code:-Core-2(Algebra)
one mark question

1. The rank of a matrix is defined as:
a) The number of rows in the matrix.
b) The number of columns in the matrix.
c) The maximum number of linearly independent rows or columns.
d) The minimum number of linearly independent rows or columns.

1. The row-rank of a matrix is:
   a) The number of rows in the matrix.
   b) The number of linearly independent rows.
   c) The sum of all row elements.
   d) The product of all row elements.
2. The column-rank of a matrix is:
   a) The number of columns in the matrix.
   b) The number of linearly independent columns.
   c) The sum of all column elements.
   d) The product of all column elements.
3. The Rank-Nullity Theorem relates the rank of a matrix to its:
   a) Nullity.
   b) Determinant.
   c) Eigenvalues.
   d) Diagonal elements.
4. For two matrices A and B, if rank(A) = m and rank(B) = n, what is the maximum possible rank of the sum A + B?
   a) m + n
   b) m
   c) n
   d) 0
5. If A and B are two matrices such that AB is defined, then which of the following is true?
   a) rank(AB) = rank(A) + rank(B)
   b) rank(AB) ≤ min(rank(A), rank(B))
   c) rank(AB) ≥ max(rank(A), rank(B))
   d) rank(AB) = rank(A) × rank(B)
6. A characteristic vector of a matrix A corresponding to a characteristic root λ is a non-zero vector that satisfies:
   a) $A\lambda = \lambda$
   b) $A + \lambda I = 0$
   c) $(A - \lambda I)v = 0$
   d) $Av = \lambda v$
7. The sum of the characteristic roots of a matrix A is equal to:
   a) The determinant of A.
   b) The trace of A.
   c) The rank of A.
   d) The eigenvalues of A.
8. If A is an n × n matrix, how many characteristic roots does it have?
   a) n
   b) 2n
   c) n^2
   d) It depends on the rank of A.
9. The Cayley-Hamilton theorem states that every matrix satisfies its own:
   a) Characteristic equation.
   b) Determinant equation.
   c) Eigenvalue equation.
   d) Trace equation.
10. Which of the following matrices satisfies the Cayley-Hamilton theorem?
    a) Any square matrix.
    b) Only diagonalizable matrices.
    c) Only invertible matrices.

d) Only symmetric matrices.

11. A quadratic form is a homogeneous polynomial of degree:
    a) 1 b) 2 c) 3 d) 4
12. A positive definite quadratic form:
    a) Takes only non-negative values for non-zero vectors.
    b) Takes only positive values for non-zero vectors.
    c) Takes both positive and negative values for non-zero vectors.
    d) Takes only negative values for non-zero vectors.
13. Which of the following matrices is triangular?
    a) A symmetric matrix b) A skew symmetric matrix c) A diagonal matrix d) An idempotent matrix
14. If A is a symmetric matrix, what can be said about its diagonal elements?
    a) They must be zero b) They can be any real numbers c) They must be positive d) They are equal
15. A matrix is said to be idempotent if:
    a) Its determinant is zero b) Its transpose is equal to its inverse c) It is equal to its square d) It has all elements equal to one
16. Which type of matrix satisfies the property $A = -A^T$?
    a) Symmetric matrix b) Skew symmetric matrix c) Diagonal matrix d) Involuntary matrix
17. Hermitian matrices are a generalization of which type of matrices? a) Diagonal matrices b) Symmetric matrices c) Skew symmetric matrices d) Orthogonal matrices
18. What is the characteristic feature of an orthogonal matrix?
    a) Its determinant is zero b) Its inverse is equal to its transpose c) It has all elements equal to one d) It is equal to its square
19. A matrix is said to be singular if:
    a) It is equal to its square b) It has all elements equal to zero c) Its determinant is zero d) It is a triangular matrix
20. Which matrix operation can be used to check if a matrix is singular?
    a) Addition b) Multiplication c) Transposition d) Determinant calculation
21. The trace of a matrix is the:
22. a) Sum of its diagonal elements b) Product of its diagonal elements c) Sum of all its elements d) Determinant of the matrix
23. An involutory matrix is a matrix that:
    a) Is equal to its square b) Has all elements equal to one c) Is orthogonal d) Has a determinant of zero
24. Which type of matrix satisfies the property $A^TA = I$?
    a) Orthogonal matrix b) Unitary matrix c) Singular matrix d) Idempotent matrix
25. A matrix is said to be nilpotent if:
    a) It has all elements equal to zero b) Its determinant is zero c) It is equal to its square d) It becomes the zero matrix when raised to a power
26. Which of the following matrices is both Hermitian and skew Hermitian?
27. a) Zero matrix b) Identity matrix c) Diagonal matrix with all real entries d) Matrix with all imaginary entries
28. Which of the following statements about singular matrices is true?
    a) Every square matrix is singular b) A matrix is singular if and only if its determinant is zero c) A matrix is singular if and only if it is symmetric d) Singular matrices are always idempotent
29. The determinant of an orthogonal matrix is:
    a) Always 1 b) Always -1 c) Always 0 d) Any real number
30. If a matrix is idempotent, then its eigenvalues are:
    a) All equal to zero b) All equal to one c) All equal to -1 d) Any real numbers

one and half mark question

31. Which type of matrix is represented by $A = A^TA$?
    a) Orthogonal matrix b) Involuntary matrix c) Unitary matrix d) Symmetric matrix
32. The sum of a matrix and its transpose is always:
    a) Symmetric b) Skew symmetric c) Diagonal d) Identity
33. Which of the following matrices is not necessarily invertible?

34. a) Idempotent matrix b) Orthogonal matrix c) Involuntary matrix d) Unitary matrix
35. If A and B are both idempotent matrices, then their product AB is:
    a) Always idempotent b) Always a zero matrix c) Always an identity matrix d) Not idempotent in general
36. If A is a unitary matrix, then its inverse is:
    a) The transpose of A b) The conjugate transpose of A c) The negative of A d) The identity matrix .
37. The eigenvalues of a diagonal matrix are:
    a) The diagonal entries of the matrix b) All equal to 1 c) All equal to 0 d) Any real numbers
38. Which of the following statements about the product of two orthogonal matrices is true?
    a) The product is orthogonal b) The product is symmetric c) The product is idempotent d) The product is diagonal.
39. If A is a singular matrix, then its inverse is:
    a) Not defined b) Equal to the transpose of A c) Equal to the adjoint of A d) Equal to the negative of A .
40. The sum of a symmetric matrix and a skew symmetric matrix is always:
    a) Symmetric b) Skew symmetric c) Diagonal d) Identity
41. Which of the following matrices is orthogonal?
    a) A matrix with all elements equal to 1 b) A matrix with all elements equal to 0 c) A matrix whose rows are orthogonal unit vectors d) A matrix whose elements are all complex numbers
42. The product of two orthogonal matrices is:
    a) Always orthogonal b) Always a zero matrix c) Always an identity matrix d) Not necessarily orthogonal in general.
43. The eigenvalues of a nilpotent matrix are:
    a) All equal to zero b) All equal to one c) All equal to -1 d) Any real numbers.
44. Which type of matrix satisfies the property $A^TA = AA^T = I$?
    a) Unitary matrix b) Involuntary matrix c) Symmetric matrix d) Skew symmetric matrix .
45. The adjoint of a matrix is also known as its:
    a) Inverse b) Conjugate c) Transpose d) Determinant .
46. If A is an idempotent matrix, then its rank is:
    a) Always 0 b) Always 1 c) Always n (where n is the size of the matrix) d) Any integer value between 0 and n.
47. The product of a matrix and its inverse is always:
    a) The identity matrix b) A zero matrix c) A diagonal matrix d) An involutory matrix .
48. Which of the following statements about singular matrices is true?
    a) Every singular matrix is symmetric b) Every non-singular matrix is idempotent c) The product of two singular matrices is always singular d) The sum of two singular matrices is always singular.
49. The eigenvalues of a unitary matrix are:
    a) Always equal to 1 b) Always equal to -1 c) Always complex conjugates of each other d) Any complex numbers with absolute value 1.
50. If A is a skew Hermitian matrix, then its diagonal elements are:
    a) Always real b) Always imaginary c) Always zero d) Any complex numbers
51. Which of the following matrices is not necessarily square?
    a) Identity matrix b) Skew symmetric matrix c) Symmetric matrix d) Unitary matrix .
52. The eigenvalues of an involutory matrix are:
    a) Always equal to 1 b) Always equal to -1 c) Always equal to 0 d) Any real numbers.
53. If A is a Hermitian matrix, then its eigenvalues are:
    a) Always real b) Always imaginary c) Always complex conjugates of each other d) Any complex numbers.
54. Which of the following statements about orthogonal matrices is true?
    a) The product of two orthogonal matrices is always orthogonal b) The sum of two orthogonal matrices is always orthogonal c) An orthogonal matrix is always idempotent d) An orthogonal matrix is always singular .
55. A matrix is said to be involutory if:
    a) It is equal to its inverse b) It is equal to its square c) It is orthogonal d) It has all elements equal to zero.
56. Which of the following matrices is a unitary matrix?
    a) A matrix with all elements equal to 1 b) A matrix with all elements equal to 0 c) A matrix whose rows are orthogonal unit vectors d) A matrix with all diagonal elements equal to 1.

57. If A is a diagonal matrix, then its trace is equal to:
    a) The sum of its diagonal entries b) The product of its diagonal entries c) The determinant of the matrix d) The rank of the matrix.
58. Which of the following matrices is not necessarily symmetric?
59. a) Identity matrix b) Skew symmetric matrix c) Orthogonal matrix d) Involuntary matrix .
60. The eigenvalues of a symmetric matrix are:
    a) Always real b) Always imaginary c) Always complex conjugates of each other d) Any complex numbers .

two mark question

61. Which of the following matrices is orthogonal?
    a) A matrix with all elements equal to 1 b) A matrix with all elements equal to 0 c) A matrix whose columns are orthogonal unit vectors d) A matrix with all diagonal elements equal to 1.
62. The product of two idempotent matrices is:
    a) Always idempotent b) Always a zero matrix c) Always an identity matrix d) Not idempotent in general.
63. If A is a skew symmetric matrix, then its eigenvalues are:a) Always real b) Always imaginary c) Always zero d) Any complex numbers .
64. The eigenvalues of a skew Hermitian matrix are:
    a) Always real b) Always imaginary c) Always complex conjugates of each other d) Any complex numbers.
65. A matrix is said to be involutory if and only if:
    a) Its determinant is zero b) It is equal to its inverse c) It is equal to its square d) It is a symmetric matrix.
66. Which of the following statements about nilpotent matrices is true?
    a) Every nilpotent matrix is singular b) A matrix is nilpotent if and only if its determinant is zero c) The product of two nilpotent matrices is always nilpotent d) The sum of two nilpotent matrices is always nilpotent.
67. What is a determinant of a matrix primarily used for?
    a) Solving differential equations b) Calculating eigenvectors c) Finding the area/volume scaling factor d) Solving systems of linear equations
68. The determinant of a 3x3 matrix requires evaluating:
    a) 6 minors b) 2 minors c) 9 minors d) 3 minors
69. If the determinant of a matrix is zero, what does that imply?
    a) The matrix is singular b) The matrix is invertible c) The matrix has linearly dependent rows or columns d) The matrix is diagonalizable
70. The determinant of a matrix changes when:
    a) Rows are multiplied by a scalar b) Columns are interchanged c) Rows are added to other **rows** d) Columns are scaled by a constant
71. The determinant of a triangular matrix is:
72. a) Always 1 b) The sum of its diagonal elements c) The product of its diagonal elements d) Equal to the product of its diagonal elements multiplied by a sign factor
73. What is the determinant of the identity matrix?
    a) 1 b) 0 c) -1 d) It depends on the order of the matrix
74. The determinant of the transpose of a matrix is:
    a) Always positive b) Equal to the determinant of the original matrix c) Equal to the negative of the determinant of the original matrix d) Always zero
75. If the determinant of a 4x4 matrix is 7, what is the determinant of its inverse?
    a) 7 b) 1/7 c) 28 d) -7
76. The determinant of a product of two matrices A and B is equal to:
    a) The determinant of A plus the determinant of B b) The determinant of A times the determinant of B c) The determinant of A multiplied by the determinant of B d) The determinant of A minus the determinant of B
77. The determinant of an orthogonal matrix is:
    a) Always 0 b) Either 1 or -1 c) Always 1 d) Always -1
78. If a matrix A is multiplied by its inverse, the result is:
79. a) The zero matrix b) The identity matrix c) A matrix with all entries equal to 1 d) The identity matrix scaled by a factor
80. What is the relationship between the determinant of a matrix and its adjoint?

a) The determinant is the sum of the entries of the adjoint. b) The determinant is the average of the entries of the adjoint. c) The determinant is the trace of the adjoint. d) The determinant is the product of the entries of the adjoint.

81. Cramer's rule is used to:
a) Calculate the rank of a matrix b) Find the eigenvalues of a matrix c) Solve systems of linear equations d) Compute the determinant of a matrix
82. In Cramer's rule, what is the numerator for finding the solution to the ith variable?
a) Determinant of the matrix A b) Determinant of the augmented matrix [A | b] c) Determinant of the matrix obtained by replacing the ith column of A with b d) Determinant of the inverse of matrix A
83. The echelon form of a matrix is obtained by:
a) Swapping rows b) Swapping columns c) Performing row operations d) Performing column operations
84. The determinant of an upper triangular matrix is equal to:
a) The product of its diagonal elements b) The sum of its diagonal elements c) The product of its diagonal elements multiplied by a sign factor d) The sum of its off-diagonal elements
85. If the determinant of a matrix is -5, what is the determinant of its transpose?
a) 5 b) -5 c) 0 d) It cannot be determined from the given information
86. The determinant of a 2x2 matrix $\begin{vmatrix} a & b \\ c & d \end{vmatrix}$ can be calculated using the formula:
a) ad - bc b) ac - bd c) $(a^2 + b^2 + c^2 + d^2)$ d)ad - bc
87. Which of the following is a property of the determinant of a product of matrices?
a) det(AB) = det(A) + det(B) b) det(AB) = det(A) * det(B) c) det(AB) = det(A) * det(B) * det(AB) d) det(AB) = det(A) - det(B)
88. The determinant of a 3x3 matrix can be found using:
a) Only row operations b) Only column operations c) Only matrix inversion d) Row operations or column operations
89. The adjoint of a matrix A is denoted by:
a) adj(A) b) inv(A) c) tr(A) d) A
90. Which property of a matrix is used to determine if it is invertible using its determinant?
a) Diagonalizability b) Non-singularity c) Orthogonality d) Symmetry

Model Question-01
GOVERNMENT AUTONOMOUS COLLEGE, PANPOSE
ROURKELA

Subject –Statistics (UG-1st year, 1st Semester). **Full MARK-60**
Paper code:-Core-1 (Algebra) **Time-3 Hrs**

Answer All Questions
The figures in right hand margin indicate marks.

1. **Answer all questions** **[1X8=8]**

(a) A zero of $x^3 + 64$ is____________?
(b) If f and g are polynomials of degrees m and n respectively, and if $h(x) = (f * g)(x)$, then the degree of h is____________?
(c) How many property must be satisfy for a non-empty set V is called a vector space.
(d) Give an example of vector space.
(e) A matrix is known as symmetric if A=__________
(f) The trace of the matrix is ______________
(g) A square matrix A=$[a_{ij}]_{nXn}$ if $a_{ij} = 0$ $for\ i > j$ then that matrix is known as ________.
(h) For matrix A if $A^T = I$, I is identity matrix then A is __________.

2. **Answer any Eight questions** **[1.5X8=12]**

(a) If one of the roots of a quadratic equation is 2+ $\sqrt{3}$ form the equation.
(b) If one root of the quadratic equation $2x^2 + kx – 6 = 0$ is 2, the value of k is_______?

(c) The roots of the quadratic equation $\frac{1}{a+b+x} = \frac{1}{a} + \frac{1}{b} + \frac{1}{x}$ is ______
(d) R is a vector space over Z (True/False)
(e) if F is afield and T is a sub field of F , then f is a vector space over T (True/False)
(f) If A is a skew-symmetric matrix, then its diagonal elements must be________
(g) The transpose of a skew-symmetric matrix is always _____________
(h) An orthogonal matrix is a matrix whose columns are____________
(i) An idempotent matrix is a matrix that ________.
(j) A square matrix whose diagonal element exit and others are zero the matrix is called as___________

3. Answer any Eight questions **[2X8=16]**

a) Define scalar matrix?
b) Define strictly upper triangular matrix?
c) If matrix A is idempotent, then its eigenvalues are.?
d) A Hermitian matrix is also known as?
e) If A is a which satisfy the property A = -A* , then it is called as _________ matrix.
f) The product of two orthogonal matrices is always.
g) An involuntary matrix is a matrix that.
h) The inverse of a unitary matrix is.
i) if A square matrix satisfy the property $AA^* = I$, then a is _______ matrix.
j) For any square matrix A and B, tra(AB) is equal to ______ (tra(A)+tra(B))

4.Answer all questions **[6X4=24]**

4) a)If the roots of the equation $x^3 + px^2 + qx + r = 0$ are in arithmetic progression, then prove that $2p^3$−9pq+27r=0

OR

b) Solve $x^4 - 4x^2 + 8x + 35 = 0$ given 2 + i 3 is a root.

P.T.O

5) a) Let S be the set of all vectors of the form {x, 2x, -3x, x} in V . Then S be a subspace of V.

OR

b) If S be any non-empty subset of a vector space V prove that [S]=S Iff S is a subspace of V.

6) a) If A, B, C are square matrices of order n such that A=B+C, BC=CB and C^2=0 then show that every +ve integer " p" , $CA^{p+1} = B^p(B + (p + 1)C)$.

OR

b) Prove that $\begin{bmatrix} \lambda & 1 \\ 0 & \lambda \end{bmatrix}^n = \begin{bmatrix} \lambda^n & n\lambda^{n-1} \\ 0 & \lambda^n \end{bmatrix}$ for n $\in N$

7)a). Solve the linear equation by Cramer's rule.

OR

b). If A be any Nilpotent matrix of order 2. Show that $A(I + A)^n = A$ for any positive integer "n"